AP Calculus AB Summer Assignment **Name:_____**

Due Date: The last class day of the second week of school.

The purpose of this assignment is to have you practice the mathematical skills necessary to be successful in AP Calculus AB. Most of the skills covered in this packet are skills from Algebra 2 and Pre-Calculus, some are skills from an introduction to the Calculus. If you need to, you may use reference materials to assist you and refresh your memory (old notes, textbooks, online resources, etc.). While the graphing calculators will be used in class, there are *no calculators allowed* on this packet. You should be able to do everything without a calculator.

AP Calculus AB is a fast paced course that is taught at the college level. There is a lot of material in the curriculum that must be covered before the AP exam in May. Therefore, we cannot spend a lot of class time re-teaching prerequisite skills. This is why you have this packet. Spend some time with it and make sure you are clear on everything covered in the packet so that you will be successful in Calculus. Of course, you are always encouraged to seek help from your teacher if necessary.

Your assignment is to do <u>at least the odd-numbered problems</u>. Do more if you feel you need the practice. This assignment is worth 50 points.

This assignment will be collected and graded the <u>last class day of the second week of school</u>. Be sure to show all appropriate work to support your answers. In addition, there may be a quiz on this material during the first quarter.

Good Luck!

Ms. Falletta

Name:

Show all work – no credit will be awarded for answers missing appropriate work. No calculators!

Section I: Algebra Review

Identify the following statements as true or false.

1. $\frac{x+y}{2}=\frac{x}{2}+\frac{y}{2}$ _____

2. $\frac{1}{p+q}=\frac{1}{p}+\frac{1}{q}$ _____

3. $\frac{2k}{2x+h}=\frac{k}{x+h}$ _____

4. $3\bullet\frac{a}{b}=\frac{3a}{b}$ _____

5. $3\bullet\frac{a+b}{c}=\frac{3a+b}{c}$ _____

6. $\sqrt{a^2+b^2}=a+b$ _____

Identify the following statements as true or false over the set of real numbers. Give a counter example for any false statement.

7. $x^3+1>x^3$ _____

8. $x^3+x>x^3$ _____

9. $x^2\geq 0$ _____

10. $x^2\geq x$ _____

11. $2x\geq x$ _____

12. $\sqrt{x}\geq 0$ _____

13. $-x\leq 0$ _____

14. $\frac{1}{x}\leq x$ _____

15. $x\leq|x|$ _____

16. Solve $xy'+y+1=y'$ for y'.

17. Solve $\ln y=kt$ for y.

16._____

17._____

18. Factor: y^3+27

19. Factor: $x^2(x-1)-4(x-1)$

18._____

19._____

Simplify each expression.

20. $\frac{\left(x^2\right)^3 x}{x^7}$ _____

21. $\sqrt{x}\bullet\sqrt[3]{x}\bullet x^{\frac{1}{6}}$ _____

22. $\frac{5(x+h)^3-5x^3}{h}$ _____

23. $\frac{3(x+h)^2-3x^2}{h}$ _____

24. $\dfrac{\dfrac{x^2-1}{x}}{\dfrac{x+1}{x^3}}$ _____

25. $\dfrac{\dfrac{1}{x}+\dfrac{4}{x^2}}{3-\dfrac{1}{x}}$ _____

26. $\dfrac{\dfrac{a}{2x+h}-\dfrac{a}{2x}}{h}$ _____

27. $\dfrac{1}{1-2a}-\dfrac{2}{1+2a}+\dfrac{6a+2}{4a^2-1}$ _____

Simplify, using factoring of binomial expressions. Leave answers in factored form.

Example:

$$\frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)} = \frac{(x+1)^2\left[(x+1)(4x-9)-(16x+9)\right]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2(4x^2-5x-9-16x-9)}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2(4x^2-21x-18)}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2(4x+3)(x-6)}{(x-6)(x+1)}$$

$$= (x+1)(4x+3)$$

28. $(x-1)^3(2x-3)-(2x+12)(x-1)^2$ _____

29. $\dfrac{(x-1)^2(3x-1)-2(x-1)\bullet 3}{(x-1)^4}$ _____

30. $\dfrac{(x-1)^3(2x-3)-(4x-1)(x-1)^2}{(x-1)^2(2x-1)}$ _____

Simplify by rationalizing the numerator.

Example:

$$\frac{\sqrt{x+4}-2}{x}=\frac{\sqrt{x+4}-2}{x}\bullet\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}=\frac{x+4-4}{x\left(\sqrt{x+4}+2\right)}=\frac{x}{x\left(\sqrt{x+4}+2\right)}=\frac{1}{\sqrt{x+4}+2}$$

31. $\dfrac{\sqrt{x+9}-3}{x}$ _____

32. $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$ _____

Solve each equation or inequality for x over the set of real numbers.

33. $2x^4+3x^3-2x^2=0$ _____

34. $\dfrac{2x-7}{x+1}=\dfrac{2x}{x+4}$ _____

35. $\dfrac{3x+5}{(x-1)(x^4+7)}=0$ _____

36. $\sqrt{x^2-9}=x-1$ _____

37. $|2x-3|=14$ _____

38. $x^2-2x-8<0$ _____

Solve each of the systems.

39. $x+y=8$
$2x-y=7$ _____

40. $y=x^2-3x$
$y=2x-6$ _____

Section II: Pre-Calculus Review

Use your knowledge of the unit circle to evaluate each of the following. Leave your answers in radical form.

41. $\sin(30^\circ)$ _____

42. $\cos\frac{2\pi}{3}$ _____

43. $\tan 45^\circ$ _____

44. $\sin\left(-\frac{\pi}{6}\right)$ _____

45. $\tan\pi$ _____

46. $\csc\frac{5\pi}{6}$ _____

47. $\cos(90^\circ)$ _____

48. $\cos\frac{3\pi}{4}$ _____

49. $\tan\frac{\pi}{6}$ _____

50. $\cos^{-1}\left(\frac{1}{2}\right)$ _____

51. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ _____

52. $\tan^{-1}(1)$ _____

Solve each trigonometric equation for $0 \le x \le 2\pi$.

53. $\sin x = \frac{\sqrt{3}}{2}$ _____

54. $\tan^2 x = 1$ _____

55. $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$ _____

56. $2\sin^2 x + \sin x - 1 = 0$ _____

For each trigonometric function identify the amplitude and any horizontal or vertical shifts from the basic function.

57. $y = \frac{1}{2}\cos\frac{x}{2} - 3$ amplitude:_____ period:_____ vertical shift:_____

58. $y = 2\sin(2x - \pi)$ amplitude:_____ period:_____ horizontal shift:_____

59. $y = \tan 3x$ period:_____

Solve each exponential or logarithmic equation.

60. $5^x = 125$ _____

61. $8^{x+1} = 16^x$ _____

62. $81^{\frac{3}{4}} = x$ _____

63. $8^{\frac{-2}{3}} = x$ _____

64. $\log_2 32 = x$ _____

65. $\log_x \frac{1}{9} = -2$ _____

66. $\log_4 x = 3$ _____

67. $\log_3(x+7) = \log_3(2x-1)$ _____

Expand each of the following using the laws of logs.

68. $\log_3 5x^2$ _____

69. $\ln \frac{5x}{y^2}$ _____

Complete each of the following using trigonometric identities and formulas.

70. $\sin\left(\frac{\pi}{2} - x\right) =$ _____

71. $\sin^2 x + \cos^2 x =$ _____

72. $\sin 2u =$ _____

73. $\tan x =$ _____

74. $1 + \cot^2 x =$ _____

75. $1 - \cos^2 x =$ _____

76. A right triangle has a base of 5 and a hypotenuse of 7. Find the height.

Section III: Graphing Review

Sketch the following functions. State the domain and range of each. Draw and label your own axes.

77. $f(x) = x$

78. $f(x) = x^2$

79. $f(x) = x^3$

80. $f(x) = |x|$

81. $f(x) = [x]$ (Greatest integer function)

82. $f(x) = \frac{1}{x}$

83. $f(x) = \sqrt{x}$

84. $f(x) = e^x$

85. $f(x) = \ln x$

86. $f(x) = \sqrt{9 = x^2}$

87. $f(x) = \sin x$

88. $f(x) = \cos x$

89. $f(x) = \tan x$

90. $f(x) = \csc x$

91. $f(x) = \sec x$

92. $f(x) = \cot x$

Section IV: Intro to Calculus Review – Limits and Derivatives

Evaluate each limit.

93. $\lim_{x\to 0} 3\cos 2x + 2$ _____

94. $\lim_{x\to 3} \frac{x^2 - 4x + 3}{x - 3}$ _____

95. $\lim_{x\to 4} \frac{\sqrt{x} - 2}{x - 4}$ _____

Use the limit definition of derivative to find the derivative of:

96. $f(x) = 4x^2 + 3x - 5$

Use the derivative rules to find the derivative of each function.

97. $f(x) = 3x^2 - 5x + 11$ _____

98. $f(x) = 5x^3(x^4 - 3x^2)$ _____